Executive Equity Mix, Earnings Manipulation, and Project Continuation

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Abstract: This paper studies the optimal design of long-term executive equity compensation when boards of directors make project termination decisions based on accounting information and executives can take costly actions to manipulate this information. The model shows that incentives for manipulation (i) persist even though the optimal contract requires the CEO to hold his equity for the long run (i.e., through long vesting periods) and (ii) decrease with a shift in emphasis from stock options towards restricted stock. In addition, the paper analyzes how the optimal equity mix and the level of manipulation change when the financial reporting environment changes. Specifically, the model predicts that the magnitude of manipulation is an inverted U-shaped function of the CEO’s opportunistic reporting discretion. Thus, in countries in which accounting standards are lax (tight) and governance controls are weak (strong), taking steps to limit reporting discretion will increase (reduce) the level of manipulation. With respect to CEO equity pay, the model predicts that firms

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place greater emphasis on restricted stock relative to stock options in countries with more lax accounting standards and weaker governance.

1 Introduction

Over the last decade, fuelled by accounting scandals in the US and Europe, executive equity compensation has come under increased public scrutiny. A widely held concern is that equity based pay causes executives to engage in accounting manipulation and fraud in an attempt to reap personal gains. The incentive to manipulate arises from the executives’ desire to cash out vested stock and options at inflated short-term prices. According to this view, the larger the executives’ equity holdings, the greater is the temptation for misreporting. However, the empirical evidence on the link between CEO equity incentives and proxies for accounting irregularities is inconclusive at best (see the discussion in Armstrong et al., 2010). Thus, whether and how equity based pay affects executives’ incentives to manipulate accounting information is still an open question.

The present paper demonstrates that there is another more subtle reason for why equity compensation can affect manipulation incentives that is unrelated to the desire to inflate short-term prices. When boards of directors rely on accounting information as a basis for decision making, executives have an incentive to manipulate this information to steer decisions in their favor. Indeed, in the model studied here, the optimal equity pay plan endogenously creates a wedge between the CEO’s and the board’s preferred decision and thus introduces a desire to misreport information.

The distinction between these two motives for manipulation (short-term price inflation versus decision distortion) is important because each can be addressed through
different mechanisms. Clearly, the desire to inflate short-term prices increases when the CEO can unload larger equity packages (e.g., Goldman and Slazek, 2006). The board can alleviate price inflation incentives either by granting smaller equity packages or by restricting the CEO’s freedom to cash out equity in the short run, for example, through long vesting periods. These remedies do not effectively address manipulation incentives that arise from the CEO’s desire to distort decision making. The decision distortion incentive will persist even when the board relies on optimal long-term contracts. In addition, the CEO’s incentive to distort decisions through manipulation does not necessarily depend on the size of the equity grant but on the mix of stock and options. The model shows that a shift in emphasis away from stock options towards restricted stock lowers incentives for misreporting and improves reporting quality.

Specifically, I consider a setting in which a board of directors has to motivate a CEO to work on a new project. The firm’s accounting system generates a publicly observable report that is informative about the future performance of the project. Based on this report, the board decides whether or not to terminate the project at an early stage. An important feature of the model is that the CEO can engage in costly manipulative activities in an attempt to distort the accounting signal in his favor. A higher level of manipulation reduces reporting quality and hence the efficiency of the board’s project continuation decision.

Consider first the optimal pay plan assuming that the level of manipulation is exogenously given. In this case, the only goal of the board is to provide effective effort incentives and the optimal contract consists exclusively of long-term vesting stock options and no restricted stock. Stock options are more effective than restricted stock because of their convex nature. Specifically, options generate a positive payoff
for the CEO only if the accounting report is favorable and the project succeeds, whereas stock has a positive value even when the report is unfavorable and the project is terminated.

A contract that consists solely of stock options is not necessarily optimal when the CEO’s manipulation incentives are taken into consideration. The reason is again the convex nature of options. When the project is terminated, options lose their value, biasing the CEO in favor of continuing the project. Given that the board makes continuation decisions based on the accounting report, the CEO is inclined to distort bad news in an attempt to avoid termination. As a consequence, stock options create an incentive to manipulate the interim accounting report despite the use of long vesting periods.

The board can alleviate the CEO’s manipulation incentive by replacing some of the stock options with restricted stock. Restricted stock is less effective in inducing effort but aligns the CEO’s incentive regarding project abandonment with those of the board (and shareholders) and thereby curbs the CEO’s desire to distort information.\(^1\) The model predicts that executive pay packages with higher stock to option ratios are associated with lower levels of accounting manipulation and hence higher reporting quality.

In principle, by choosing an equity mix with a sufficiently strong emphasis on restricted stock relative to options, the board is able to fully eliminate manipulation incentives while keeping up incentives to work hard. However, such a contract is not optimal because it is too costly to shareholders.

An important factor that determines both the equilibrium level of manipulation

\(^1\)In the Appendix, I solve for the optimal unrestricted contract and show that it can be replicated by a mix of restricted stock and stock options.
and the optimal design of the equity contract is the CEO’s opportunistic reporting discretion. When the CEO has limited (broad) reporting discretion, it is relatively difficult (easy) to successfully manipulate accounting information. In practise, managerial reporting discretion is a function of the financial reporting environment (such as accounting standards, its legal enforcement, and litigation), the firm’s corporate governance structures that relate to reporting oversight (such as financial expertise on the board and the audit committee, and the quality of the auditor), and firm characteristics that determine the difficulty with which outsiders can oversee reporting (such as firm size and the complexity of operations). To analyze the impact of the CEO’s reporting discretion on the optimal equity pay plan and the equilibrium level of manipulation, two cases need to be considered.

Consider first an environment in which the CEO’s reporting discretion is limited. In this case, regardless of the incentive pay plan, the board is little concerned about potential manipulation. As a result, the board optimally ignores the manipulation control problem (and the associated distortion of the project termination decision) and designs an incentive contract that deals effectively with the effort control problem. Such a contract consists exclusively of stock options. In this environment, a further reduction in reporting discretion leads to less manipulation and better project continuation decisions, consistent with conventional views. The reduction in manipulation, in turn, relaxes the effort incentive problem and reduces the number of options required to induce effort.

Consider now an environment in which the CEO has broad reporting discretion. If the board relies solely on stock options as a means to induce effort, the CEO’s manipulation attempts will be severe. Successful manipulation not only distorts the board’s termination decision but also leads to excessive CEO compensation. Con-
sequently, the board finds it optimal to adjust the equity pay package to combat manipulation and the optimal contract consists of a mix of options and stock. In this case, a marginal reduction in the CEO’s reporting discretion does not lower but actually increases the equilibrium level of manipulation. The broad intuition for this result is as follows. When the board designs the incentive plan it balances (i) the cost of inducing effort with (ii) the benefit of efficient project abandonment. When the CEO’s reporting discretion declines, adjusting the pay plan to mitigate manipulation becomes less effective and the board shifts attention away from (ii) toward (i). As a result, the optimal contract places greater emphasis on options relative to stock, which leads to both a lower cost of contracting and a higher manipulation incentive.

Coupling these two cases shows that the level of manipulation in organizations is an inverted U-shaped function of the CEO’s reporting discretion. The model predicts that in firms and countries in which accounting standards are lax (tight) and governance controls are weak (strong), taking steps to limit opportunistic reporting discretion through improvements in accounting standards or governance increases (reduces) the magnitude of manipulation. In addition, the model predicts that in countries with lax accounting standards and weak governance, firms put greater emphasis on restricted stock relative to stock options than in countries with tight standards and strong governance.

In contrast to the present paper, most of the existing work on accounting manipulation focuses on settings in which CEO pay can only be linked to an interim earnings report or interim stock price but not to the actual long-term outcome or long-term stock price (e.g., Dye, 1988; Feltham and Xie, 1992; Goldman and Slezak, 2006; Crocker and Slemrod, 2006; Laux and Laux, 2009). In these settings, it is impossible to encourage effort and eliminate manipulation at the same time. A no-
ticeable exception is Evans and Sridhar (1996) who allow for long-term contracting and analyze conditions under which it is optimal to design an incentive plan that simultaneously eliminates manipulation and motivates effort. The present paper is also related to Ewert and Wagenhofer (2005), who study the effects of tighter accounting standards on incentives for accounting and real earnings management in a setting in which executive compensation is exogenous. The current model contributes to this stream of literature by studying the determinants and impacts of the equity mix in long-term pay plans when boards rely on accounting information for decision making.

Section 2 outlines the model. Section 3 studies the optimal contract assuming that the level of manipulation is fixed. Section 4 analyzes how changes in the equity mix affect incentives for effort and manipulation. Section 5 determines the optimal contract and conducts a comparative static analysis. Section 6 discusses the empirical implications of the model and Section 7 concludes.

2 The Model

Consider a setting with three risk-neutral parties: shareholders, the board of directors, and the CEO. The board of directors represents the interests of shareholders and is responsible for designing the incentive contract for the CEO and deciding whether or not to terminate a project at an early stage.

In the beginning of the game, the board hires the CEO to work on the new project and offers him a pay plan. The specifics of this pay plan are outlined below. After the CEO signed the contract, he chooses an unobservable action, \( a = \{a_L, a_H\} \), with \( a_H > a_L \), that positively influences the future outcome of the project. There are two states of the world, \( S = S_L \) and \( S = S_H \). With some abuse of notation, the action
a represents the probability that the good state, $S_H$, occurs. Thus, when the CEO chooses action $a_i$, the state of the world is good, $S = S_H$, with probability $a_i$ and bad, $S = S_L$ with probability $(1 - a_i)$. The state realization is not publicly observable and hence cannot be used for contracting. The CEO’s private cost associated with action $a$ is $v(a)$, where $v(a_H) = K$ and $v(a_L) = 0$. To avoid trivial solutions, I assume that the board always wishes to induce the high action.

The state $S$ determines whether the investment project should be terminated or continued. If continued, the future outcome of the project, denoted $x$, is either $x = X > 0$ (success) or $x = 0$ (failure). The state affects the project’s success probability as follows. If $S = S_j$, the probability of success is $p_j$, where $j = L, H$ and $p_H > p_L$. If the project is abandoned, then the outcome is a constant, $x = L > 0$, with $p_H X > L > p_L X$. Thus, the Pareto efficient decision is to continue the project if the state is good, $S = S_H$, and to abandon it otherwise. If no additional information about the state is available, I assume it is optimal to continue the project; that is, 

$$(a_H p_H + (1 - a_H) p_L) X \geq L.$$ 

The accounting system produces a publicly observable report, $R \in \{R_L, R_H\}$, that is informative about the state $S$. In the absence of manipulation, the accounting report is perfectly informative about $S$ in the sense that $R = R_j$ when $S = S_j$, where $j = L, H$. However, the manager can exert manipulation effort, $m \in [0,1]$, in an attempt to distort the report. If the manager is successful in manipulating the report, the accounting system produces $R = R_j$ when in fact $S = S_i$, with $i \neq j$ and $i, j \in \{L, H\}$. Due to the proximity of the project, the manager observes the state prior to his manipulation decision. Given manipulation effort $m$, the report is distorted with probability $qm$ and undistorted with probability $(1 - qm)$, where $q \in [0,1]$ is a measure of the effectiveness of manipulation. Choosing a positive level
of manipulation, $m$, involves a personal cost of $0.5km^2$ for the CEO, with $k \geq 0$. The disutility of manipulation captures potential litigation risk, loss of reputation, and psychic costs.

The two key parameters that determine the CEO’s financial reporting discretion are the marginal effectiveness and cost of manipulation, $q$ and $k$, respectively. As the parameter $k$ increases and/or the parameter $q$ declines, it becomes more difficult and costly for the CEO to successfully manipulate the report. These parameters capture factors such as the tightness of accounting standards and its legal enforcement, the quality of auditors, financial expertise on the board and the audit committee, and the difficulty with which outsiders can oversee financial reporting. In the comparative static analysis in Section 5, I focus on the effects of changes in the parameter $k$. Focusing on the parameter $1/q$ would not qualitatively change the results.

After the accounting report, $R$, is issued, the board decides whether or not to continue the project. The board is unable to precommit to a specific termination policy up front and hence abandons/continues the project whenever this is ex post optimal. As discussed in detail later, the CEO will never manipulate the report downwards. Thus, if the accounting report is unfavorable, $R = R_L$, the board knows that the state is bad and abandons the project because $p_LX < L$. However, if the report is favorable, $R = R_G$, the board is unsure whether the state is good or whether the report has been manipulated. Nevertheless, given the above assumptions, it is efficient to continue the project if the report is favorable. To see this, note that the probability that $S = S_H$ conditional on observing a favorable report, $R = R_H$, is

$$\rho(m*) \equiv \Pr(S_H|R_H, m*) = \frac{a_H}{a_H+(1-a_H)qm*},$$

where $m*$ is the level of manipulation in equilibrium. Given $R = R_H$, the expected cash flow associated with continuing the
project is
\[ V^C = (\rho(m^*)p_H + (1 - \rho(m^*))p_L) X, \]
which can be rewritten as
\[ V^C = \left( \frac{a_H (p_H - p_L)}{a_H + (1 - a_H)m^*q} + p_L \right) X. \]

For \( R = R_H \), it is optimal to continue the project if \( V^C \geq L \). This condition is always implied by \( (a_H p_H + (1 - a_H)p_L) X \geq L \) and \( qm^* \leq 1 \).\(^2\)

In the beginning of the game, the board offers the CEO an incentive pay plan. The CEO is protected by limited liability in the sense that payments to the CEO must be nonnegative. The reservation utility of the CEO is normalized to zero. It is without loss of generality to assume that there is one issued share of stock, which is held by initial shareholders. The contract takes the form \( C = (\alpha, \beta_S, \beta_O, E) \), where \( \alpha \) is a fixed salary; \( \beta_S \) and \( \beta_O \) specify the number of restricted stock and stock options awarded to the CEO in the beginning of the game; and \( E \) is the exercise price of the stock options. Due to the limited liability assumption it is always optimal to set the fixed salary to zero, \( \alpha = 0 \).

Focusing on the equity pay plan \( C \) is without loss of generality because there is no other more elaborate contract that can yield a higher payoff to shareholders. To show this formally, I derive the optimal unrestricted contract in the Appendix, in which pay can be directly linked to cash flows, the accounting report, and the termination decision, and show that this contract can be replicated by equity plan \( C \).\(^3\)

\(^2\)I assume that in equilibrium the CEO’s compensation is not too large relative to the project’s cash flows such that the compensation contract has no effect on the board’s continuation/termination decision. This is the case if the cost of effort, \( K \), (which determines the magnitudes of CEO payments) is not too large relative to the cash flows \( X_H \) and \( L \).

\(^3\)The board could expand the set of contractible variables by asking the CEO to communicate
Note that the optimal equity pay plan has two additional features (see the Appendix for proofs). First, the equity compensation must be long-term; that is, the contract must prevent the CEO from unloading his equity before the firm’s final cash flows are realized. This can be achieved by using equity with long vesting periods. Otherwise, if the CEO is allowed to cash out his equity immediately after the accounting report is issued, his incentive to manipulate the report will be stronger and the effort incentive problem will be more severe. Second, to provide effective effort incentives, it is optimal to set the exercise price of the stock options sufficiently high, $E \in [L, X]$, such that the options have a positive value if and only if the project is continued and successful. This is the case, for example, if the board awards options with an exercise price that equals the stock price at the grant date (i.e., at-the-money options). Note that any exercise price that lies in the range $[L, X]$ is optimal. Intuitively, although changes in $E$ change the value of the CEO’s stock options, this effect can always be neutralized by adjusting the size of the option award, $\beta_O$.

The CEO’s payoff is now as follows. If the project is abandoned, the CEO’s stock options become worthless and the value of his equity depends exclusively on the size of the stock grant and is given by $\beta_S L + \beta_O \max(L - E, 0) = \beta_S L$. If the project is continued, the value of the CEO’s equity is given by $\beta_S x + \beta_O \max(x - E, 0)$, which translates into $\beta_S X + \beta_O (X - E) > 0$ when the project succeeds and zero when it

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fails.

The goal of the board is to blend stock and option grants to optimize the convexity of the pay package. When the contract consists exclusively of restricted stock, the CEO’s pay is linear in the outcome $x$. By adding options, the board increases the convexity of the pay plan. Of course, equity pay is not the only mechanism that can be used to optimize pay convexity. I focus on equity instruments as a means to implement the optimal incentive contract because equity is ubiquitous in executive pay arrangements and I am interested in the question of how a change in the mix of different equity components changes CEO behavior.

The timing of events is as follows:

Stage 1: The board offers contract $C$ and the CEO decides whether or not to accept the offer. After accepting the offer, the CEO makes an effort choice, $a$.

Stage 2: The CEO observes the state $S$ and chooses the level of manipulation, $m$.

Stage 3: The accounting report $R$ is issued and the board terminates the project if the report is unfavorable and continues it otherwise.

Stage 4: Final cash flows are realized.

### 3 Inducing Effort

In this section, I treat the level of manipulation, $m$, as exogenously given and focus on the optimal equity contract that induces effort. Suppose that the CEO only engages in manipulation when the bad state is realized but not when the good state is realized (as shown in Section 4 this is indeed the case). Given this strategy, the CEO’s expected utility if the state is good is

$$U_{H}^{CEO} \equiv p_{H} (\beta_{O} (X - E) + \beta_{S} X), \tag{1}$$
and his utility if the state is bad is

\[ U_L^{CEO}(m) = qmp_L (\beta_O (X - E) + \beta_S X) + (1 -qm)\beta_S L - 0.5km^2. \tag{2} \]

The CEO chooses to work hard if the following effort incentive constraint is satisfied:

\[ U_H^{CEO} - U_L^{CEO}(m) \geq K/ (a_H - a_L). \tag{3} \]

Both, stock options and restricted stock provide the CEO with incentives to work. For convenience, we can rewrite the effort constraint (3) to obtain the size of the option grant, \( \beta^*_O(\beta_S, m) \), that is required to induce effort as a function of the stock award:

\[ \beta^*_O(\beta_S, m) = \frac{K/ (a_H - a_L) - 0.5km^2 - ((p_H X - L) + qm (L - p_L X)) \beta_S}{(p_H - qmp_L) (X - E)}. \tag{4} \]

Note that

\[ \frac{\partial \beta^*_O(\beta_S, m)}{\partial \beta_S} = -\frac{(p_H X - L) + qm (L - p_L X)}{(X - E)(p_H - qmp_L)} < 0. \]

For the remainder of this paper I focus on equity pay plans that are incentive compatible; that is, satisfy (4). Given (4), when the number of stock awarded to the CEO increases, the required number of options declines and the stock to option ratio, \( \beta_S/\beta_O \), increases. There are an infinite number of contracts that are incentive compatible and each contract can be characterized by a specific stock to option ratio, \( \beta_S/\beta_O \). But only one contract is associated with the lowest cost of compensation. In the present setting, given that the level of manipulation is fixed, stock options are strictly more effective in inducing effort than restricted stock. To see this note that the contract provides the strongest incentives when the wedge between the CEO’s payoffs \( U_H^{CEO} \) and \( U_L^{CEO}(m) \) is maximized. This is the case when the board relies exclusively on stock options. The advantage of options is that they have a positive value only if the accounting report is favorable and the project succeeds in the long
run, whereas shares of restricted stock have value even when reported performance is poor and the project is abandoned.

**Lemma 1** Assuming that the level of manipulation, \( m \), is fixed, the optimal equity contract consists solely of stock options; \( \beta_O^* > 0, \beta_S^* = 0 \).

For the discussions in later sections it is useful to elaborate on this result. The expected cost of CEO compensation can be expressed as

\[
V = a_H U_H^{CEO} + (1 - a_H) \left( U_L^{CEO}(m) + 0.5km^2 \right), \tag{5}
\]

where \( U_H^{CEO} \) and \( U_L^{CEO}(m) \) are as in (1) and (2). Recalling that the size of the option award, \( \beta_O^*(\beta_S, m) \), is determined by (4), taking the first derivative of (5) with respect to \( \beta_S \) yields:

\[
\frac{dV}{d\beta_S} = \frac{p_H(1 - qm)}{p_H - qmp_L} L > 0. \tag{6}
\]

Condition (6) confirms that implementing effort through an equity package that is more stock and less option heavy increases the cost of CEO compensation. However, this effect is weaker when the level of manipulation is larger:

\[
\frac{d^2V}{d\beta_S dm} = -\frac{qp_H (p_H - p_L)}{(p_H - qmp_L)^2} L < 0.
\]

The intuition for this result is as follows. The benefit of stock options over restricted stock is that options have no value when the report is unfavorable and the project is terminated. But a higher level of manipulation implies that the CEO is more likely to avoid an unfavorable report, so the advantage of stock options over stock declines with \( m \). In the extreme, when \( qm = 1 \), the two equity components are equally effective in inducing effort. However, as shown next, the level of manipulation is not a constant but a function of the mix of stock and options.
4 Controlling Effort and Manipulation

Consider now the original setting in which the level of manipulation is the CEO’s choice variable. I first analyze the effects of changes in the equity pay mix on effort and manipulation and then present the optimal contract in Section 5.

Suppose the CEO observes the good state, $S = S_H$. In the absence of manipulation, the report is favorable, $R = R_H$, and the project is continued. The CEO will only have an incentive to manipulate the report downwards if the value of his equity in case of an unfavorable report, $\beta_S L$, exceeds the expected pay in case of a favorable report, $p_H (\beta_O (X - E) + \beta_S X)$; that is, if $-\beta_S (p_H X - L) > p_H \beta_O (X - E)$. Given $\beta_S, \beta_O \geq 0$, this inequality is never satisfied. Thus, for $S = S_H$, the CEO chooses $m^* = 0$ and the CEO’s expected utility is as in (1).

Suppose now the CEO observes the bad state, $S = S_L$. In this case, the CEO chooses the level of manipulation that maximizes (2). The first-order condition for an optimal level of $m$ can be written as

\[ p_L \beta_O (X - E) - \beta_S (L - p_L X) = m^* k/q. \]  

(7)

Substituting the effort incentive constraint (4) into (7) yields the level of manipulation as a function of $\beta_S$:

\[ \frac{p_L K/ (a_H - a_L) - (0.5 p_L km^* + \beta_S L (p_H - p_L))}{(p_H - q p_L m^*)} = m^* k/q. \]  

(8)

Note that the equilibrium manipulation choice, $m^*$, determined by (8) is unique.\(^\text{5}\)

\(^4\)Note that the second-order condition for a maximum is satisfied.

\(^5\)There exists two values for $m$ that satisfy (8) but one value exceeds one and hence lies outside the permitted region.
As discussed in the previous section, for a fixed level of $m$, the contract that most effectively induces effort consists exclusively of stock options. Recall from Section 2 that the options do not vest and thus cannot be exercised until the project’s final cash flows are realized. Despite the long vesting terms, the value of the CEO’s stock options is linked to the interim accounting report. This link arises because the board bases its termination decision on accounting information and the stock options lose their value when the project is abandoned. As a result, stock options provide the CEO with incentives to distort bad news in an attempt to avoid project abandonment.

The board can counteract manipulation incentives by awarding the CEO restricted stock in lieu of stock options. Restricted stock aligns the CEO’s incentives regarding project abandonment with those of shareholders and thereby curbs the desire to distort information.

Formally, using (8), we have

$$\frac{dm^*}{d\beta_S} = -\frac{(p_H - p_L) q}{(p_H - qp_L m^*) k} L < 0,$$

which confirms that a shift from options to stock lowers manipulation incentives. Note that this shift in equity components has a stronger reducing effect on $m^*$ when the equilibrium level of manipulation is high than when it is low:

$$\frac{d^2 m^*}{d\beta_S dm^*} = -\frac{(p_H - p_L) p_L q^2}{(p_H - qp_L m^*)^2 k} L < 0.$$

**Proposition 1** When the board implements effort through a contract with a higher stock to option ratio, the CEO’s incentive to engage in manipulation declines.

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6 Applying the implicit function theorem on (8), we have

$$\frac{dm^*}{d\beta_S} = -\frac{L (p_H - p_L) / (p_H - qp_L)}{(p_H - qmp_L) (\beta_L(p_H - p_L))^{1/2} - k/q}.

Substituting (8) into (9) yields (10) after some rearranging.
For the discussion in Section 5 it is useful to analyze how a change in the stock to option ratio affects the expected compensation cost $V$ (given in (5)). Recalling that the size of the option grant is determined by (4) and the level of manipulation is determined by (8), the first derivative of (5) with respect to $\beta_S$ can be written as

$$\frac{dV}{d\beta_S} = \left(1 - (2 - a_H)qm^* \frac{p_H - p_L}{p_H - qP_Lm^*}\right) L.$$  \hspace{1cm} (12)

In contrast to the previous section, a shift from options to stock can either increase or reduce the cost of CEO compensation. The sign and strength of this relation depends on the equilibrium level of manipulation, $m^*$, which is itself a function of the equity mix and the reporting discretion, represented by $k$.

**Lemma 2** i) There exists a unique threshold, $m_T \in (0, 1)$, such that

$$\frac{dV}{d\beta_S} > 0 \text{ if } m^* < m_T \text{ and } \frac{dV}{d\beta_S} < 0 \text{ if } m^* > m_T,$$

ii) $\frac{d^2V}{d\beta_Sdm} < 0$.

Implementing effort through an equity contract with a higher stock to option ratio has two effects on the compensation cost $V$. First, as outlined in the previous section, holding $m$ fixed, an increase in $\beta_S/\beta_O$ directly increases $V$ because stock is less effective in inducing effort than options. This effect is weaker when the equilibrium level of manipulation is larger. Second, a higher stock to option ratio reduces the level of manipulation. This effect is stronger when the equilibrium level of manipulation is higher. The reduction in manipulation, in turn, lowers the expected cost of compensation.

Consequently, starting with an environment that induces a low level of manipulation ($m^* < m_T$), the first effect dominates the second effect and an increase in the
stock to option ratio leads to an increase in $V$. This positive relation becomes weaker when the equilibrium level of manipulation increases. Starting with an environment that induces a relatively high level of manipulation ($m^* > m_T$), the second effect dominates the first effect and an increase in $\beta_S/\beta_O$ reduces $V$. This negative relation becomes stronger when the level of manipulation further increases.

5 Optimal Contract and Comparative Statics

This section establishes the optimal incentive contract. The board’s objective is to choose a $(\beta_O, \beta_S)$ combination that maximizes

$$U^{Board} = a_H p_H X + (1 - a_H) (qm(p_L X) + (1 - qm)L) - V,$$

subject to the manipulation constraint (7), the effort constraint (4), and the nonnegativity constraints $\beta_O, \beta_S \geq 0$.

In principle, the board can design an incentive plan that induces effort and, at the same time, eliminates manipulation. The most cost effective equity mix that implements $(a = a_H, m = 0)$ is given by

$$\beta_S L = \frac{p_L K}{(a_H - a_L)(p_H - p_L)}, \quad (13)$$

$$\beta_O (X - E) = \frac{(L - p_L X) K}{(a_H - a_L)(p_H - p_L) L}. \quad (14)$$

The fact that it is possible to eliminate manipulation does not mean that it is indeed optimal for the board to do so.

When designing the optimal contract, the board focuses on the two goals of minimizing the cost of inducing effort and implementing efficient abandonment decisions. The question is how does a change in the equity mix affect these two goals.
The effect of a change in the equity mix on the efficiency of the abandonment decision is clear. If the board implements effort through an equity contract with a higher stock to option ratio, the CEO’s manipulation attempt declines, which increases the accuracy of the accounting report. More accurate reports, in turn, enable the board to make better project abandonment decisions. The effects of changes in the equity mix on the cost of the incentive system are more subtle and have already been explored to some extent in the previous section.

To determine the optimal contract it is necessary to distinguish between two different cases.

**Limited reporting discretion:** Consider first an environment in which the CEO has limited reporting discretion ($k > k_T$, where $k_T$ is defined in the Appendix). The CEO’s manipulation opportunities are restricted and the level of manipulation is small even when the board relies exclusively on stock options as a means to induce effort. A shift away from stock options toward restricted stock further reduces manipulation incentives and thereby improves the board’s abandonment decision. However, we know from Lemma (2) that for low levels of $m^*$ an increase in the stock to option ratio strongly increases the cost of compensation, $V$. I show in the Appendix that for $k > k_T$, the increase in the compensation cost outweighs the benefits of improved abandonment decisions. As a result, the board optimally ignores the CEO’s manipulation attempts and offers the equity pay plan that is most effective in inducing effort. This contract consists exclusively of stock options.

**Proposition 2** Suppose that the CEO has limited reporting discretion ($k > k_T$). The optimal equity pay plan consists of stock options and no restricted stock with

$$\beta^*_O (X - E) = 0.5 \frac{-km^*2 + 2K/(a_H - a_L)}{PH - qpm^*} \text{ and } \beta^*_S = 0,$$

(15)
and the equilibrium level of manipulation, $m^*$, satisfies

$$(K/(a_H - a_L) + 0.5km^*) p_L/p_H - m^*k/q = 0.$$  

In this situation, a further reduction in reporting discretion (increase in $k$) reduces the level of manipulation and improves reporting quality, consistent with conventional views. In addition, restricting reporting discretion reduces the rent the CEO can obtain when the state is bad and hence relaxes the effort incentive problem. As a consequence, the level of stock options required to motivate high effort declines with $k$. Finally, when opportunistic reporting discretion declines, shareholder value increases. The next proposition summarizes the results.

**Proposition 3** Suppose that $k > k_T$. When $k$ increases,

(i) the level of manipulation, $m^*$, declines, leading to more efficient project termination decisions,

(ii) the size of the stock option grant, $\beta_O^*$, declines, and

(iii) shareholder value, $U_{Board}$, increases.

**Broad Reporting Discretion:** Consider now an environment in which the CEO has broad reporting discretion, $k < k_T$. If the board relies exclusively on stock options to induce effort, the CEO has a strong incentive to manipulate. A move away from stock options toward restricted stock lowers the level of manipulation and improves the project abandonment decision. In addition, we know from Lemma (2) that for high values of $m^*$, a shift toward restricted stock either mildly increases or even reduces the cost of the incentive pay plan (depending on the specific value of $m^*$). As

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7 Using (4) and (8), and keeping $\beta_S = 0$ fixed, it holds that
d\[ \frac{\partial \beta_O}{\partial k} = \frac{\partial \beta_O(m,k,\beta_S)}{\partial k} + \frac{\partial \beta_O(m,k,\beta_S)}{\partial m} \frac{dm}{dk} = -0.5m^* \frac{m^2}{(p_H - qmp_L)(X - E)} < 0 \] where \( \frac{\partial \beta_O(m,k,\beta_S)}{\partial m} \) = 0.
a consequence, for \( k < k_T \), the board finds it optimal to curb the CEO’s manipulation incentive and the optimal contract consists of a mix of stock options and restricted stock. The optimal equity mix balances the cost of the pay plan with the benefits of improved decision making.

**Proposition 4** Suppose the CEO has broad reporting discretion, \( k < k_T \). The optimal equity pay plan consists of stock options and restricted stock with

\[
\beta_S^* L = \left( \frac{p_L \left( \frac{K}{(a_H - a_L)} + 0.5km^2 \right) - \frac{k}{q} m^* p_H}{(p_H - p_L)} \right), \quad (16)
\]

\[
\beta_O^* (X - E) = \frac{(L - p_L X) \left( \frac{K}{(a_H - a_L)} + 0.5km^2 \right) + \frac{k}{q} m^* (p_H X - L)}{(p_H - p_L) L}, \quad (17)
\]

and the equilibrium level of manipulation, \( m^* \), satisfies

\[
m^* = \max \left\{ 0, \frac{-q \left( (1 - a_H) (p_H - p_L) (L - p_L X) + p_H k/q \right)}{k \left( (1 - a_H) (p_H - p_L) + p_H \right)} \right\}.
\]

A reduction in managerial discretion (increase in \( k \)) has a direct and an indirect effect on the equilibrium level of manipulation. The direct effect is similar to the one discussed in the previous case: all else equal, restricting discretion leads to less manipulation. The indirect effect follows because the board will respond to a change in managerial discretion by adjusting the incentive pay plan. This adjustment not only completely eliminates the direct effect but provides the CEO with further incentives to manipulate. Thus, reducing reporting discretion actually increases the level of manipulation and hence reduces reporting quality. The broad intuition for this result is as follows. When the CEO has less reporting discretion, adjustments in the incentive plan have a smaller impact on the CEO’s manipulation choice and hence on the efficiency of the abandonment decision. Thus, the trade-off between reducing the cost of CEO compensation and improving abandonment decisions shifts in favor of the
former. As a consequence, the board optimally reduces the stock to option ratio in the equity contract, which provides more effective effort incentives but also strengthens the CEO’s incentive to manipulate. Despite the increase in manipulation, firm value increases with reduced reporting discretion because the board is able to induce effort at a lower cost.

**Proposition 5** Suppose that \( k < k_T \). When \( k \) increases,

(i) the level of manipulation, \( m^* \), increases, leading to less efficient project termination decisions,

(ii) the size of the stock option grant, \( \beta_O^* \), increases; and the size of the restricted stock grant, \( \beta_S^* \), decreases, and

(iii) shareholder value, \( U^{Board} \), increases.

Propositions (3) and (5) directly lead to the next corollary.

**Corollary 1** (i) The level of manipulation is an inverted U-shaped function of managerial reporting discretion, \( k \).

(ii) The optimal size of the stock option grant, \( \beta_O^* \), is an inverted U-shaped function of reporting discretion, \( k \). The optimal size of the restricted stock grant, \( \beta_S^* \), is weakly decreasing in \( k \).

6 Discussion and Empirical Predictions

6.1 Effects of Equity Incentives on Manipulation

In recent years, equity based compensation has been criticized for encouraging executives to engage in accounting manipulation and fraud. A motive for manipulation
arises because executives can reap personal gains by boosting accounting numbers and cashing out equity at inflated prices. However, this argument applies only for vested equity that the CEO is free to unload (e.g., Core, 2010). Thus, the general concern is that awarding executives larger equity packages that can be unloaded in the short run leads to larger levels of manipulation and fraud. The empirical evidence on this relation has produced inconclusive results (e.g., Bergstresser and Philippon, 2006; Burns and Kedia, 2006; Erickson et al., 2006; Johnson et al., 2009; Armstrong et al., 2010).

The current model shows that even long-term equity pay that cannot be cashed out in the short run can cause executives to manipulate interim accounting reports. The desire to distort accounting information arises because the board bases its project termination decision on the accounting report and the optimal equity pay plan creates a wedge between the CEO’s and the board’s preferred decision. Manipulation is costly for shareholders not only because it increases expected CEO pay but also because it leads to inefficient project termination decisions. The incentive to distort accounting information is determined by the mix of the CEO’s equity package. When the equity contract puts greater emphasis on options relative to stock, the CEO has a stronger preference in favor of project continuation and hence has a stronger incentive to manipulate unfavorable reports to steer the decision in his favor. Thus, the model predicts that in firms in which the stock to option ratio of executives’ equity pay plans is smaller, the level of earnings management is larger (Proposition 1).

6.2 Reporting Discretion and Manipulation

An important factor that determines both the equilibrium level of manipulation and the optimal choice of the equity mix is the CEO’s opportunistic reporting discretion,
captured by the variable \( k \) in the model. In practice, managerial reporting discretion is a function of the financial reporting environment (such as accounting standards, legal enforcement of standards, and litigation), the firm’s corporate governance structures that relate to reporting oversight (such as financial expertise on the board and the audit committee, and the quality of the auditor), and other firm characteristics that determine the difficulty with which outsiders can oversee financial reporting (such as firm size and complexity of operations). There is a large and growing number of empirical studies analyzing the effects of accounting standards or governance mechanisms on reporting quality (e.g., Dechow et al., 1996; Beasley, 1996; Klein, 2002; Farber, 2005; Larcker et al., 2007; Barth et al., 2008; Jeanjean and Stolowy, 2008). The empirical evidence so far, however, has not produced a consistent set of results.

The present model shows that changes in opportunistic reporting discretion affect incentives for accounting manipulation not only directly but also indirectly through the optimal choice of the equity pay mix. For this reason, and as demonstrated in Corollary 1, the level of manipulation in firms is not a linear but an inverted U-shaped function of managerial reporting discretion. This result has the following two empirical implications.

First, the model suggests that the effect of changes in the firm’s financial reporting environment on the prevalence of manipulation depends on whether executives initially have broad or limited reporting discretion. Specifically, in countries in which accounting standards are lax and governance controls are weak, taking steps to restrict discretion by improving accounting standards or governance actually increases the level of manipulation (Propositions 5). In contrast, in countries in which accounting standards are tight and governance is strong, further reductions in opportunistic reporting discretion reduces the level of manipulation (Proposition 3).
Second, the model suggests that the observed reporting quality in corporations is not a good indicator of the quality of the financial reporting environment. To elaborate, consider empirical studies that observe that the transition from local GAAP to IFRS reporting is associated with a reduction in earnings manipulation. This finding does not necessarily imply that IFRS is more effective in restricting managerial reporting discretion than local GAAP. Rather, the opposite could be true. The reduction in the magnitude of manipulation then occurs (despite the manager’s greater freedom to misreport) because the board understands the CEO’s reporting freedom and adjusts the equity pay mix accordingly (i.e., moves away from stock options toward restricted stock).

### 6.3 Determinants of Equity Pay Mix

The firm’s financial reporting environment is an important determinant of the optimal design of equity compensation. The model predicts that in countries in which accounting standards are lax and governance controls are weak, firms put greater emphasis on restricted stock relative to options than in countries in which standards are tight and governance is strong (Corollary 1). In addition, assuming that directors and auditors have a harder time overseeing financial reporting in larger firms with more complex business operations, the model predicts that the emphasis on restricted stock relative to options is greater in larger than in smaller firms.

Empirical studies that analyze the effects of corporate governance structures on executive pay design so far have focused on the pay-performance sensitivity of executive compensation (e.g., Hartzell and Starks, 2003 and Fahlenbrach, 2009). To the best of my knowledge there are no empirical studies that examine the link between governance measures and the mix of equity pay.
7 Conclusion

This paper studies the optimal long-term equity pay plan when executives can engage in costly activities to distort accounting information. Equity pay can affect manipulation incentives for different reasons. While observers and regulators have focused on manipulation incentives that arise from executives’ desire to unload vested equity at inflated short-term prices, I focus on another motive for manipulation that persists even when the CEO is required to hold his equity for the long run. When the board relies on accounting information for decision making such as terminating projects, the CEO has an incentive to engage in manipulation to tilt the decision in his favor. The mix of stock and options in the equity pay package determines the CEO’s preferences regarding project continuation/termination and hence his incentive to manipulate accounting information. The model predicts that a shift in emphasis away from options toward stock is associated with lower levels of manipulation. Although it is possible to choose an equity mix that fully eliminates manipulation and simultaneously encourages effort, this is not optimal because it is too costly. Thus, in equilibrium, the board tolerates manipulation to some extent.

The financial reporting environment is an important determinant of both the optimal equity pay plan and the magnitude of manipulation. The analysis shows that the level of manipulation in organizations is an inverted U-shaped function of the CEO’s opportunistic reporting discretion. The model predicts that in firms and countries in which accounting standards are lax (tight) and governance controls are weak (strong), taking steps to limit reporting discretion increases (reduces) the magnitude of manipulation and hence reduces (increases) reporting quality. In addition, the model predicts that in countries in which accounting standards are lax and governance is
weak, firms place greater emphasis on stock relative to options in executive compensation arrangements than in countries in which standards are tight and governance is strong.

Appendix

Proof of Equation (12).

Substituting (1) and (2) into (5) yields after some rearranging:

\[ V = (a_H p_H + (1 - a_H) q m^* p_L) (\beta^*_O(\beta_S) (X - E) + \beta_S X) + (1 - a_H) (1 - q m^*) \beta_S L, \]

(18)

where \( \beta^*_O(\beta_S) \) and \( m^* \) are determined by (4) and (8) respectively. Taking the first derivative of (18) with respect to \( \beta_S \) yields

\[ \frac{dV}{d\beta_S} = q (1 - a_H) (p_L \beta^*_O(\beta_S) (X - E) - \beta_S (L - p_L X)) \frac{dm^*}{d\beta_S} \\
+ (a_H p_H + (1 - a_H) q p_L m^*) \left( \frac{d\beta^*_O(\beta_S)}{d\beta_S} (X - E) + X \right) \\
+ (1 - a_H) (1 - q m^*) L. \]

(19)

Substituting (7) into (19) yields

\[ \frac{dV}{d\beta_S} = (1 - a_H) k m^* \frac{dm^*}{d\beta_S} + (a_H p_H + (1 - a_H) q m^* p_L) \left( \frac{d\beta^*_O(\beta_S)}{d\beta_S} (X - E) + X \right) \\
+ (1 - a_H) (1 - q m^*) L. \]

(20)

Using (4), we have

\[ \frac{d\beta^*_O(\beta_S, m)}{d\beta_S} = \frac{\partial \beta_O(\beta_S, m)}{\partial \beta_S} + \frac{\partial \beta_O(\beta_S, m)}{\partial m} \frac{dm^*}{d\beta_S} < 0, \]

(21)

where

\[ \frac{\partial \beta_O(\beta_S, m)}{\partial \beta_S} = -\frac{(p_H X - L) + q m (L - p_L X)}{(p_H - q m p_L) (X - E)} < 0, \]

27
and

\[
\frac{\partial \beta_O(\beta_S, m)}{\partial m} = q p_L K / (a_H - a_L) - 0.5 p_L k m^2 - \beta_S L (p_H - p_L) - (p_H - q p_L m) m k / q (X - E) (p_H - q p_L m)^2.
\]

(22)

Substituting (8) into (22) yields

\[
\frac{\partial \beta_O(m, \beta_S)}{\partial m} = 0.
\]

Substituting (21) and (10) into (20) yields (12).

**Proof of Lemma 2.**

Using (12), observe that for \( m^* = 0 \), we have \( dV / d\beta_S = L > 0 \) and for \( q m^* = 1 \) we have \( dV / d\beta_S = -L (1 - a_H) < 0 \). Further, note that

\[
\frac{d^2 V}{d \beta_S dm} = -q p_H (2 - a_H) L \frac{p_H - p_L}{(p_H - q m p_L)^2} < 0.
\]

Hence, there exists a unique threshold level of manipulation, \( m_T \), below which the relation between \( \beta_S \) and \( V \) is positive.

**Proof of Propositions 2 and 4.**

Consider a general contract of the form \( C^G = (W_L, W_H, w_L, w_H) \). The payments \( W_L \) and \( W_H \) are short-term rewards that are linked to the realization of the interim accounting report. If \( R = R_H \), the CEO obtains \( W_H \) and if \( R = R_L \), he obtains \( W_L \). The payments \( w_H \) and \( w_L \) are long-term rewards that are linked to the final outcome of the project if it is continued. If \( x = X \), the CEO obtains \( w_H \) and if \( x = 0 \), he obtains \( w_L \). Note that there is no need to link CEO pay directly to the termination/continuation decision because pay is already linked to the accounting report, which determines the termination decision.

Consider first the CEO’s manipulation choice and suppose the CEO observes the high state \( S = S_H \). In the absence of manipulation, the report is favorable, \( R = R_H \),
and the project is continued. The CEO will only have an incentive to manipulate the report downwards if the pay in case of a low report, \( W_L \), exceeds the expected pay in case of a high report, \( W_H + p_H w_H + (1 - p_H)w_L \). As will be shown later, this is never the case in equilibrium; that is, in the optimal solution it always holds that \( W_H + p_H w_H + (1 - p_H)w_L \geq W_L \). Thus, for \( S = S_H \), the CEO chooses \( m^* = 0 \). The CEO’s expected utility in case of a high state realization is therefore given by

\[
U_{CEO}^H \equiv (W_H + p_H w_H + (1 - p_H)w_L).
\] (23)

Suppose now the CEO observes the low state, \( S = S_L \). The CEO has an incentive to manipulate the report upwards whenever the expected pay in case of a high report exceeds the expected pay in case of a low report, \( W_H + p_L w_H + (1 - p_L)w_L > W_L \). In this case, the CEO chooses the level of manipulation that maximizes

\[
U_{CEO}^L(m) \equiv qm(W_H + p_L w_H + (1 - p_L)w_L) + (1 - qm)W_L - 0.5km^2.
\] (24)

The first-order condition for an optimal level of \( m^* \) satisfies

\[
(W_H + p_L w_H + (1 - p_L)w_L) - \frac{k}{q}m^* = W_L.
\] (25)

The second-order condition for a maximum is satisfied since \( -\frac{k}{q} < 0 \).

Given the CEO’s manipulation strategy, the CEO chooses to work hard \( (a = a_H) \) if the following effort incentive constraint is satisfied:

\[
U_{CEO}^H - U_{CEO}^L(m^*) \geq K/(a_H - a_L).
\] (26)

Using (25), the effort incentive constraint (26) can be rewritten as

\[
(W_H + p_H w_H + (1 - p_H)w_L) - (W_L + 0.5km^2) \geq K/(a_H - a_L).
\] (27)
The board’s objective is now to choose \((w_L, w_H, W_L, W_H)\) that maximizes

\[
U^{\text{Board}} = a_{H} p_{H} X + (1 - a_{H}) (q_{m} (p_{L} X) + (1 - q_{m}) L)
\]

\[-a_{H} (W_{H} + p_{H} w_{H} + (1 - p_{H}) w_{L})
\]

\[-(1 - a_{H}) (q_{m} (W_{H} + p_{L} w_{H} + (1 - p_{L}) w_{L}) + (1 - q_{m}) W_{L}) ,
\]

subject to the manipulation constraint (25), the effort constraint (26), and the non-negativity constraints \(w_{H}, w_{L}, W_{H}, W_{L} \geq 0\).

The Lagrangian of the board’s optimization problem \((P)\) is as follows:

\[
G = a_{H} p_{H} X + (1 - a_{H}) (q_{m} (p_{L} X) + (1 - q_{m}) L)
\]

\[-a_{H} (W_{H} + p_{H} w_{H} + (1 - p_{H}) w_{L})
\]

\[-(1 - a_{H}) (q_{m} (W_{H} + p_{L} w_{H} + (1 - p_{L}) w_{L}) + (1 - q_{m}) W_{L})
\]

\[+ \lambda \left((W_{H} + p_{H} w_{H} + (1 - p_{H}) w_{L}) - K / (a_{H} - a_{L})\right)
\]

\[+ \lambda \left( - (q_{m} (W_{H} + p_{L} w_{H} + (1 - p_{L}) w_{L}) + (1 - q_{m}) W_{L} - 0.5 km^{2})\right)
\]

\[+ \mu \left((W_{H} + p_{L} w_{H} + (1 - p_{L}) w_{L}) - m k / q - W_{L}\right), \quad (28)
\]

where \(\lambda\) is the Lagrangian multiplier associated with the effort incentive constraint (26) and \(\mu\) is the multiplier associated with the manipulation constraint (25).

The necessary conditions for a solution to \((P)\) include:

\[
\frac{\partial G}{\partial w_j} \leq 0, \quad w_j \geq 0, \quad \text{and} \quad \frac{\partial G}{\partial w_j} w_j = 0 \quad \text{for} \quad j = L, H,
\]

\[
\frac{\partial G}{\partial W_j} \leq 0, \quad W_j \geq 0, \quad \text{and} \quad \frac{\partial G}{\partial W_j} W_j = 0 \quad \text{for} \quad j = L, H,
\]

\[
\frac{\partial G}{\partial m} = 0.
\]

There are two possible solutions to problem \((P)\).
Case 1: Suppose that in the optimal solution, it holds that \( w_H > 0 \) and \( W_L > 0 \). This implies that \( \frac{dG}{dw_H} = \frac{dG}{dW_L} = 0 \), which yields

\[
\begin{align*}
\lambda &= \frac{a_H p_H + p_L (1 - a_H)}{p_H - p_L}, \\
\mu &= -p_H \frac{1 - qm}{p_H - p_L}.
\end{align*}
\]

(29)  \hspace{1cm} (30)

Using (29) and (30), it can be shown that \( \frac{\partial G}{\partial w_H} < -1 \) and \( \frac{\partial G}{\partial w_L} < -1 \). Thus, it is optimal to set \( W_H = w_L = 0 \).

The optimal payments \( w_H > 0 \) and \( W_L > 0 \) are determined by (25) and (26), which yields

\[
\begin{align*}
W_H^*(m^*) &= \frac{K}{(a_H - a_L) - m^* k/q + 0.5 km^2} / (p_H - p_L), \\
W_L^*(m^*) &= \frac{p_L (K / (a_H - a_L) + 0.5 km^2) - m^* p_H k/q}{(p_H - p_L)}.
\end{align*}
\]

(31)  \hspace{1cm} (32)

Substituting \( W_H = w_L = 0 \), (31), and (32) into \( \frac{\partial G}{\partial m} = 0 \) and solving for \( m \) yields

\[
m^* = \max \left\{ 0, \frac{-q (1 - a_H) (p_H - p_L) (L - p_L X) + p_H k/q}{k ((1 - a_H) (p_H - p_L) + p_H)} \right\}.
\]

(33)

In the optimal solution, \( W_L^*(m^*) \geq 0 \) must be satisfied. Note that \( W_L^*(m^*) \) is decreasing in \( m^* \). Hence, \( W_L^*(m^*) \geq 0 \) is satisfied if and only if \( m^* \) is smaller than a threshold level denoted, \( m_A \), which is determined by

\[
W_L^*(m_A) = \frac{K / (a_H - a_L) + 0.5 km_A^2) p_L - m_A p_H k/q}{(p_H - p_L)} = 0.
\]

(34)

Case 2: Suppose now that in the optimal solution it holds that \( w_H > 0 \) and \( W_L = 0 \). Hence, \( \lambda \) and \( \mu \) are determined by \( \frac{dG}{dw_H} = 0 \) and \( \frac{dG}{dm} = 0 \). Let \( \lambda^S \) and \( \mu^S \) denote the solution to \( \frac{dG}{dw_H} = 0 \) and \( \frac{dG}{dm} = 0 \). Substituting \( \lambda = \lambda^S \), \( \mu = \mu^S \), (25), and \( W_L = 0 \) into \( \frac{dG}{dW_H} \) and \( \frac{dG}{dW_L} \) shows after some rearranging that \( \frac{dG}{dW_H} < 0 \) and \( \frac{dG}{dW_L} < 0 \), which implies that it is optimal to set \( W_H = 0 \) and \( w_L = 0 \).
Using \( W_L = w_L = W_H = 0 \), the optimal pay \( w_H \) and the equilibrium level of manipulation are determined by (27) and (25) and given by

\[
\begin{align*}
 w_H^*(m^*) &= \left( \frac{K}{(a_H - a_L)} + 0.5km^2 \right) / p_H, \quad (35) \\
(K / (a_H - a_L) + 0.5km^2) p_L / p_H - m^*k / q &= 0. \quad (36)
\end{align*}
\]

Setting \( W_L = 0 \) is only optimal if \( \frac{dG}{dW_L} \leq 0 \). Substituting \( \lambda = \lambda^S, \mu = \mu^S, (25), \) and \( w_L = W_H = 0 \) into \( \frac{dG}{dW_L} \) and rearranging yields

\[
\frac{dG}{dW_L} = \frac{q (p_H - p_L) (1 - a_H) (km + q (L - Xp_L)) - p_H k (1 - qm)}{(p_H - qp_Lm) k}. \quad (37)
\]

Condition \( \frac{dG}{dW_L} \leq 0 \) is satisfied if and only if

\[
m^* \leq -\frac{q (1 - a_H) (p_H - p_L) (L - p_LX) + p_H k / q}{k ((1 - a_H) (p_H - p_L) + p_H)}. \quad (38)
\]

\textbf{When is each case relevant:} For convenience, let \( m_1^* \) and \( m_2^* \) denote the equilibrium levels of manipulation for case 1 and case 2, which are determined by (33) and (36), respectively. Note that the right hand side of (38) equals \( m_1^* \) and that \( m_2^* \) equals \( m_A \) determined in (34). Thus, case 1 applies if \( m_1^* < m_2^* \) and case 2 applies if \( m_1^* > m_2^* \). Further, as shown in the proof of Propositions (3) and (5), it holds that \( dm_1^*/dk > 0, dm_1^*/dq < 0, dm_2^*/dk < 0, \) and \( dm_2^*/dq > 0 \). Thus, there exists a unique threshold, \( k_T \), determined by \( m_1^*(k_T) = m_2^*(k_T) \), such that for all \( k < k_T \) case 1 is the relevant case and for all \( k > k_T \) case 2 is the relevant case. Alternatively, letting \( Q = 1/q \), there exists a unique threshold, \( Q_T \), determined by \( m_1^*(Q_T) = m_2^*(Q_T) \), such that for all \( Q < Q_T \) case 1 is the relevant case and for all \( Q > Q_T \) case 2 is the relevant case.

\textbf{Optimal Equity Contract:} I show next that the equity contract \( C \) introduced in Section 2 can replicate the optimal unrestricted contract derived in this Appendix. In
the optimal unrestricted contract, CEO pay is always tied to the project’s long-term results if it is continued; that is, \( w_L = 0 \) and \( w_H > 0 \). To replicate \( w_L = 0 \), the equity contract has to ensure that the CEO holds his stock and options until final cash flows, \( x \), are realized. This can be achieved, for example, by offering equity with long vesting periods. Otherwise, if the CEO is free to unload his holdings in the short-run, he can reap a positive payoff that is independent of the project’s actual long-term cash flows, and hence contradicts \( w_L = 0 \).

In case 2, the optimal unrestricted contract sets \( W_L = 0 \) and \( w_H > 0 \). To replicate these payments, the exercise price of the options must be sufficiently high to ensure that stock options have no value when the report is unfavorable and the project is abandoned; that is, \( E \geq L \). Otherwise, for \( E < L \), the equity plan cannot implement \( W_L = 0 \) and \( w_H > 0 \).

Given the equity contract \( C \) with \( E \geq L \) and long vesting periods, the CEO’s payoffs are as follows. If the accounting report is unfavorable, the project is abandoned and the value of the CEO’s equity is \( \beta_S L + \beta_O \max(L - E, 0) = \beta_S L \). If the report is favorable, the project is continued and the value of the CEO’s equity is \( \beta_S X + \beta_O (X - E) \) if the project is successful and zero otherwise.

Consider the optimal contract in case 1. To implement the optimal payments \( w_H > 0 \) and \( W_L = 0 \), defined in (31) and (32), the board sets \( \beta_S > 0 \) and \( \beta_O > 0 \) such that the two equations \( W_L = \beta_S L \) and \( w_H = \beta_S X + \beta_O (X - E) \) are satisfied. The optimal contract then consists of a mix of stock and options where the values of \( \beta_S \) and \( \beta_O \) are as given in (16) and (17) in Proposition 4. Consider now the optimal contract in case 2. To implement \( w_H > 0 \) and \( W_L = 0 \), where \( w_H \) is defined in (35), the board sets \( \beta_S = 0 \) and \( \beta_O = w_H / (X - E) \). The optimal contract then consists solely of stock options and the optimal value of \( \beta_O \) is as given in (15) in Proposition
2.

Proof of Propositions 3 and 5.

Case 1: Suppose first that \( k < k_T \). In this case, the level of manipulation is determined by (33). It is straightforward to show that
\[
\frac{dm^*}{dk} = \frac{q(p_H - p_L)(1 - a_H)(L - p_LX)}{k^2((1 - a_H)(p_H - p_L) + p_H)} > 0, \tag{39}
\]
\[
\frac{dm^*}{dq} = \frac{- (1 - a_H)(p_H - p_L)(L - p_LX) - p_Hk/q^2}{k((1 - a_H)(p_H - p_L) + p_H)} < 0, \tag{40}
\]
\[
\frac{d(qm^*)}{dq} = \frac{-2q(1 - a_H)(p_H - p_L)(L - p_LX)}{k((1 - a_H)(p_H - p_L) + p_H)} < 0. \tag{41}
\]

The optimal number of restricted stock and stock options awarded to the CEO are determined by (16) and (17). It holds that
\[
\frac{d\beta^*_S}{dk} = - \frac{p_H - 0.5qplm^*m^*}{L(p_H - p_L)} \frac{k}{q} \frac{k}{L(p_H - p_L)} \frac{dm^*}{dk} < 0,
\]
\[
\frac{d\beta^*_O}{dk} = \frac{(L - p_LX)0.5m^2 + \frac{1}{q}(p_HX - L)m^*}{(X - E)(p_H - p_L)L} + \frac{(L - p_LX)km^* + \frac{k}{q}(p_HX - L)dm^*}{(X - E)(p_H - p_L)L} > 0,
\]
and
\[
\frac{d\beta^*_S}{dq} = \frac{m^*p_H}{q(p_H - p_L)q} \frac{k}{q} \frac{k}{L(p_H - p_L)} \frac{dm^*}{dq} > 0,
\]
\[
\frac{d\beta^*_O}{dq} = \frac{-m^*(p_HX - L)k/q^2}{L(p_H - p_L)} + \frac{(L - p_LX)km^* + (p_HX - L)k/q dm^*}{(X - E)(p_H - p_L)L} < 0.
\]

I show next that firm value is increasing in \( k \) and \( 1/q \). Using the Lagrangian of the board’s optimization problem (see 28), we have
\[
\frac{dG}{dk} = \lambda (0.5m^2) - \mu m/q > 0,
\]
where \( \lambda > 0 \) and \( \mu < 0 \) are determined by (29) and (30).

Using (28), and the optimal payments \( W^*_H = w^*_L = 0 \) and \( w^*_H > 0 \) and \( W^*_L > 0 \) determined in (31) and (32), we obtain
\[
\frac{dG}{dq} = - \frac{m}{q^2} (q^2(L - p_LX)(1 - a_H) + qmk(1 - a_H) - \mu k + \lambda qkm) < 0,
\]
where \( \lambda > 0 \) and \( \mu < 0 \) are given by (29) and (30).

**Case 2:** Suppose now that \( k > k_T \). In this case, the equilibrium level of manipulation is determined by (36). It is straightforward to show that

\[
\frac{dm^*}{dk} = \frac{1}{k} \left( \frac{qp_L 0.5m^{*2} - p_H m^*}{p_H - qp_L m^*} \right) < 0, \tag{42}
\]

\[
\frac{dm^*}{dq} = \frac{q (p_H - qp_L m^*)}{p_H m^*} > 0, \tag{43}
\]

\[
\frac{d(qm^*)}{dq} > 0. \tag{44}
\]

The optimal number of stock options awarded to the CEO is determined by (15).

Using (36), it can be shown that

\[
\frac{d\beta^*_Q}{dk} = -\frac{0.5m^{*2}}{(X - E) (p_H - qp_L m^*)} < 0,
\]

\[
\frac{d\beta^*_Q}{dq} = \frac{k}{q (X - E) (p_H - qp_L m^*)} > 0.
\]

I show next that firm value is increasing in \( k \) and \( 1/q \). Using the Lagrangian of the board’s optimization problem (see 28), we have

\[
\frac{dG}{dk} = \lambda \left( 0.5m^{2} \right) - \mu \left( m/q \right).
\]

Recall that \( \lambda > 0 \) and \( \mu < 0 \) are determined by \( \frac{dG}{dw_H} = 0 \) and \( \frac{dG}{dm} = 0 \). Using (35) and (36), the optimal payment \( w_H \) can be written as \( w_H = mk / (p_L q) \). Using the optimal payments \( w_H = mk / (p_L q) \) and \( W_L = w_L = W_H = 0 \), we obtain

\[
\lambda = \frac{q^2 p_L (L - p_L X) (1 - a_H) + 2km p_L (1 - a_H) + k a_H p_H}{k (p_H - qp_L)} > 0 \tag{45}
\]

and

\[
\mu = - (q (L - p_L X) + km) (1 - a_H) \frac{q}{k} < 0, \tag{46}
\]

implying that \( \frac{dG}{dk} > 0 \).
Using (28), and the optimal payments \( W_H = w_L = W_L = 0 \) and \( w_H = mk/(p_L q) \), we obtain
\[
\frac{dG}{dq} = -\frac{m}{q^2} (qmk (1 - a_H) + q^2 (L - p_L X) (1 - a_H) - \mu k + \lambda q km) < 0,
\]
where \( \lambda > 0 \) and \( \mu < 0 \) are determined by (45) and (46).

References


